Unit - VI
Transients

Transient state :- The behavior of the voltage (or) current when it is changed from one state to another state is called a transient state.
Transient time :- The time taken for the circuit to change from one state to another state is called transient time.

Even in Transient conditions, the circuit should satisfy Kirchoff's Laws. The network equations for a circuit containing energy storage elements obtained by applying Kirchoff's Laws are linear integral differential equations with constant coefficients. The solution of these differential equations represents the response of the circuit. The solution of differential equation contains two parts one is complementary (or) transient part. Another part is particular integral (or) steady-state solution response.

First order differential equations :-

first order circuit, during its transient state of operation is governed by a first order linear differential equation and in reality the first order circuit contains resistance and one inductor (or) one capacitor ie; the resistance and one energy storage component.

First order circuit equation is

$$\frac{dy}{dt} + ay = bx$$

where x = Input
y = output
a & b = Constants

Superposition and linearity both are applicable in first order passive circuits. The natural response of the first order circuit is obtained when the governing equation of the circuit as its transient state is

$$\frac{dy}{dt} + ay = 0$$

This is homogenous first order differential equation

The solution for such equation is $y = ce^{-at}$

where c = Constant

If evaluated using Initial Conditions, the solution 'c' becomes a particular solution. At $t=0$, if $y=y_0$, then the natural response of the circuit is

$$y_c = y_n = y_0 e^{-at} \quad \text{for } t>0$$

where $t=0$, y at $t_0\,(=y_0)$ becomes c in the general solution $y=ce^{-at}$

when x (input) is specified, it is called forcing function and the solution y is called the forced response (or) particular integral y_p).

The Complete solution is

$$y = y_p + y_c$$

$$= y_p + y_0 e^{-at} \quad \text{for } t>0$$

$$\text{(or)} \quad y = y_p + ce^{-at}$$

c is determined with given Initial Conditions

If the first order differential equation is non-homogeneous then

$$\frac{dy}{dt} + ay = bx = Q$$

where Q is either a function of independent variable (or) Constant

$$\therefore y = e^{-at}\int Q e^{at}\,dt + ce^{-at}$$

$$= y_p + y_c$$

where particular Integral $y_p = e^{-at}\int Q e^{at}\,dt$

and Complementary function $y_c = ce^{-at}$

If Q is Constant

$$y = e^{-at}\, Q \cdot \frac{e^{at}}{a} + c\,e^{-at}$$

$$= \left(\frac{Q}{a} + ce^{-at}\right) \Rightarrow y = \frac{Q}{a} + c\,e^{-at}$$

Second order differential equation :-

A second order circuit contains two independent energy storage elements with (or) without additions to resistance.

A second order differential equation as

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = 0$$

A, B, C are constants

The general solution of this second order differential equation is

$$y = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

where C_1 and C_2 are constants & α_1 & α_2 are the roots of the characteristic equation

$$AD^2 + BD + C = 0 \qquad \text{where } D = \frac{d}{dt}$$

$$\therefore \alpha_1, \alpha_2 = \frac{-B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC}$$

If $\alpha_1 = \alpha_2$ the roots are said to be respective type and the genaral solution is

$$y = c_1 e^{\frac{-B}{2A} \cdot t} + k_2 e^{-\frac{B}{2A} \cdot t}$$

Example: Consider the second order (differential) circuit containing R, L & C elements for convenience we assume constants A, B and C of the second order differential equation as

$$A = 1, \; B = 2\alpha \text{ and } C = \omega_0^2$$

where α = damping factor and ω_0 = frequency of oscillation

The homogeneous second order differential equation becomes

$$\frac{d^2y}{dt^2} + 2\alpha \frac{dy}{dt} + \omega_0^2 y = 0$$

The characteristic equation

$$D^2 + 2\alpha D + \omega_0^2 = 0$$

$$(D+\alpha)^2 + \omega_0^2 - \alpha^2 = 0$$

The roots of the above equation are D_1 and D_2 are called natural frequencies. Depending on the relative values of α and ω_0 there are 3 different cases for the roots.

i) $\alpha > \omega_0$ [overdamped condition]

$$D_1 = -\alpha + (\alpha^2 - \omega_0^2)^{1/2} = -S_1$$

$$D_2 = -\alpha - (\alpha^2 - \omega_0^2)^{1/2} = -S_2$$

S_1 & S_2 being real and +ve.

The natural response is $y_n = k_1 e^{-S_1 t} + k_2 e^{-S_2 t}, \; t > 0$

(ii) $\alpha < \omega_0$ [underdamped condition]

Let $\omega_0^2 - \alpha^2 = \omega_d^2$

The roots are complex conjugate with negative real part

$$D_1 = -\alpha + j\omega_d \; ; \; D_2 = -\alpha - j\omega_d$$

where ω_d = Damping frequency

The natural response

$$y_n = e^{-\alpha t}[k_1 \cos\omega_d t + k_2 \sin\omega_d t)$$

(iii) $\alpha = \omega_0$ [critically damped condition]

$(D+\alpha)^2 = 0$; $D = -\alpha = -\omega_0$

[Some times D is represent 'p' & D_1 and D_2 represents P_1, P_2]

Two roots D_1 & D_2 coincide

The natural response is

$$y_n = (k_1 + k_2 t)\, e^{-\alpha t}, \; t > 0$$

k_1 and k_2 are determined from initial conditions.

usually the values of y and $\frac{dy}{dt}$ at $t = 0^+$.

Natural Response :-

Forced Response :-

Initial Conditions :-

Initial conditions are those conditions that exist in the circuit immediately after switching operation. At $t=0$, one (or) more switches are operated which disturbs the equilibrium of the circuit.

The initial conditions depend on the past history of the network prior to $t=0^-$ (just before closing switch is operated at $t=0$) and the network structure immediately after closing the switch ($t=0^+$).

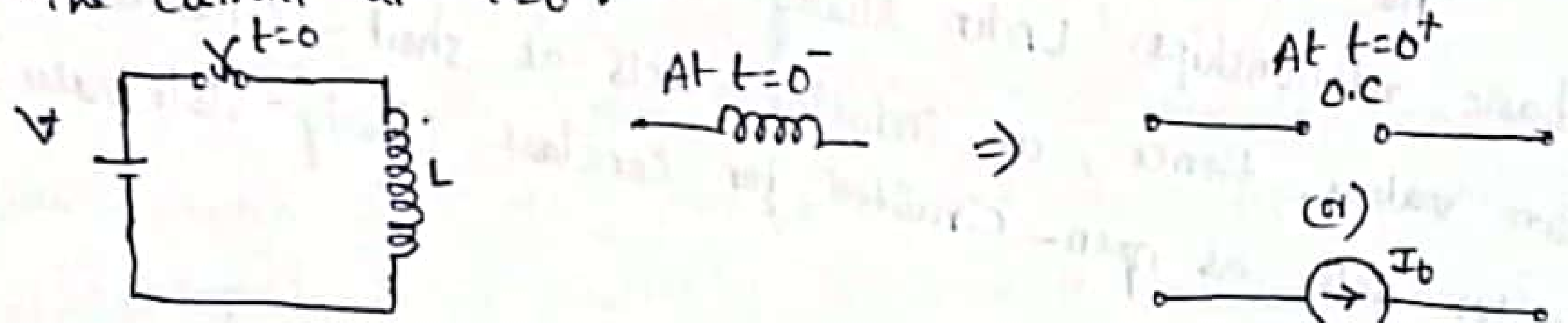
The values of capacitor voltage and inductor currents at the reference instant $t=0^-$, must be known. After switching, new currents and voltages may appear in the network at $t=0^+$ as a result of the initial

...citor voltages and the initial inductor current. The evaluation of all voltages and currents and their derivative at $t=0^+$ i.e; after switching constitutes the evaluation of initial conditions. (3)

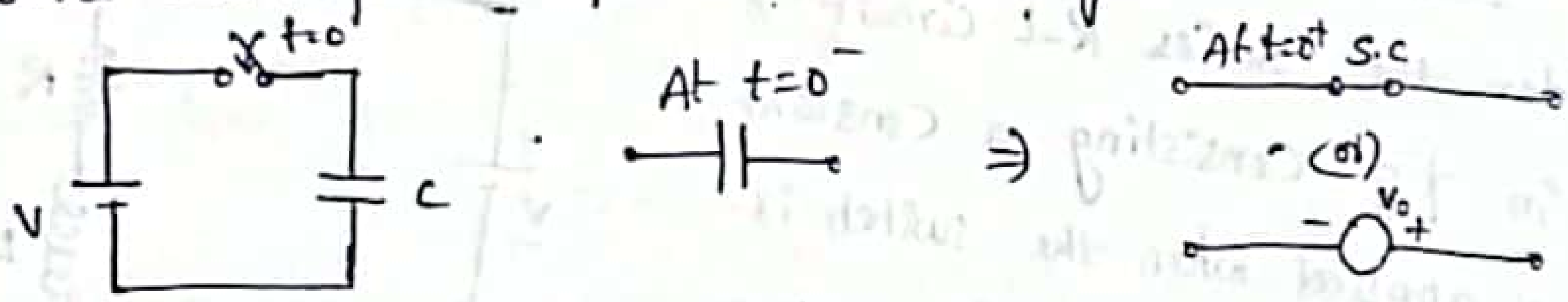
Procedure: Initial conditions for the Transient networks are evaluated by

→ The equivalent network of the network under consideration is drawn at $t=0^+$.

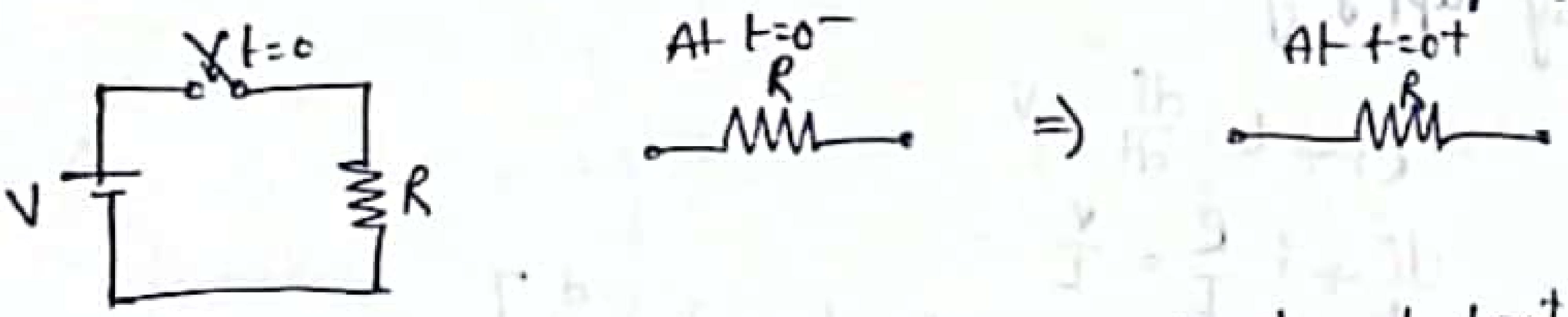
→ If an inductor is present, then it is replaced by an open circuit. It can be replaced by a constant source whose magnitude equals the current at $t=0^-$.



→ If the network contains a capacitor, then it is replaced by a short circuit is the equivalent circuit. It can also be replaced by a voltage source (V_0)., if the capacitor were charged at $t=0^-$.

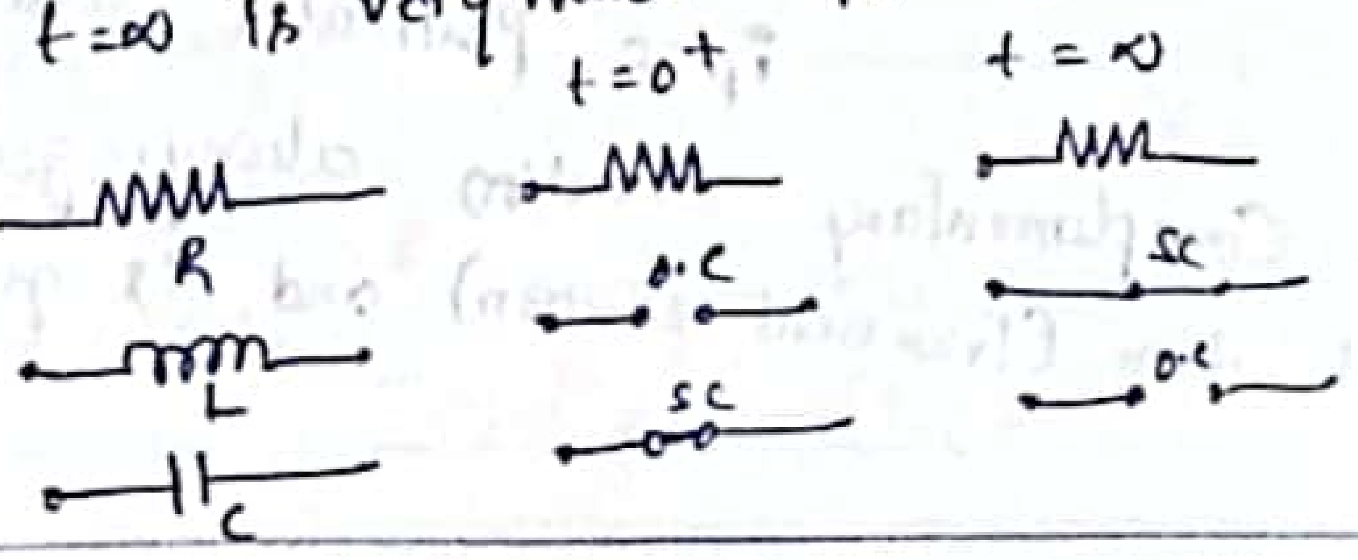


→ If the network contains a resistor remain unchange from $t=0^-$ to $t=0^+$.



The governing equations of the network at $t=0^+$ are obtained by using either KCL (or) KVL with the help of these equations, $\frac{di}{dt}$ and $\frac{dv}{dt}$ can be evaluated.

If the network remains in a particular energy state for a considerable time, and then the state of the network is changed at $t=0$, then the condition, $t=\infty$ is very much helpful for determining the network conditions.



Final steady state Conditions :-

Similar Conditions for the R, L, C in initial Condition may obtained for final conditions in the network. In which

→ All Excitation is provided by source which reduce to zero value for large 't' (or) by initial capacitor voltage (or) inductor current.

→ Networks in which final value for voltage and current is a constant.

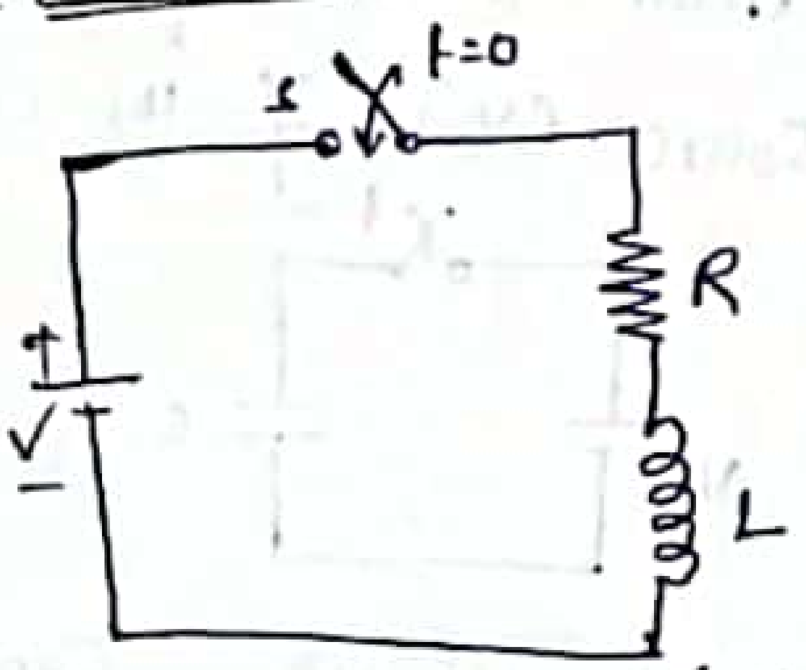
The final steady-state equivalent networks are derived from basic relationships. Under steady state condition, the derivatives have zero values. Hence, an inductor acts as short-circuited and a capacitor acts as open-circuited for constant steady-state value.

DC Transients :-

DC Excitation (First order Circuit) :-

R-L Series Circuit

Consider the series R-L circuit is shown in fig. consisting a constant voltage V applied when the switch is closed. By applying KVL for all $t \geq 0$



we get $$Ri + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{V}{L}$$

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L} \quad \text{---(1)} \qquad \left[\because \text{where } D = \frac{d}{dt}\right]$$

Comparing eqn (1) with non-homogeneous differential equation $(D+P)x = k$ and forced response is obtained from its solution, the solution is

where $P = \frac{R}{L}$, $x = i$, $k = \frac{V}{L}$

$$i = i_c + i_p$$

where i_c = Complementary function

i_p = Particular solution

Complementary function always goes to zero. Value in a relatively short time (Transient solution) and is given $i_c = C e^{-(R/L)t}$

where c = Constant

Particular solution i_p provides steady state response

$$i_p = e^{-(R/L)t} \int e^{(\frac{R}{L})t} \left(\frac{V}{L}\right) dt$$

$$= e^{-(R/L)t} \cdot \frac{e^{(R/L)t}}{\frac{R}{L}} \cdot \frac{V}{L}$$

$$= \frac{L}{R} \cdot \frac{V}{L} = \frac{V}{R}$$

Hence the net solution 'i' is

$$i = i_c + i_p = c\,e^{-(R/L)t} + \frac{V}{R} \rightarrow (2)$$

An inductance, due to its 'electrical inertia' does not allow sudden change of current through it and hence at current through inductor just before switching is same to the current just after the switching.

$$i(0^-) = i(0^+)$$

However, before switching there is no current through the inductor and at time $t = 0^+$

$$i(0^+) = 0$$

with the initial condition eq^n (2). becomes

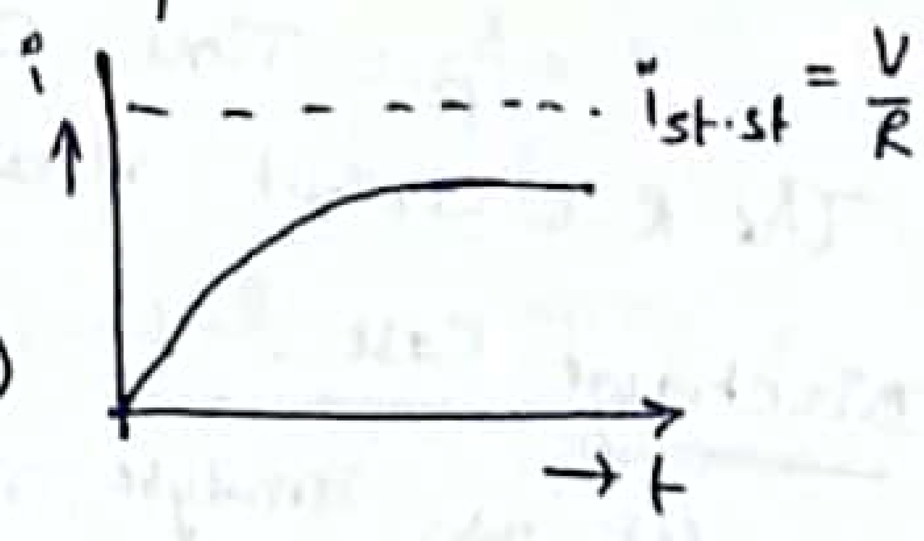
$$0 = c\,e^{-(R/L)t} + \frac{V}{R}$$

$$c = -\frac{V}{R} \qquad (\because t=0 \text{ and } e^0 = 1,\ i(0)=0)$$

$$\therefore\ i = -\frac{V}{R}\,e^{-(R/L)t} + \frac{V}{R}$$

$$i = \frac{V}{R}\left[1 - e^{-(R/L)t}\right] \rightarrow (3)$$

From the above expression exponential rise of current & charging the inductor.

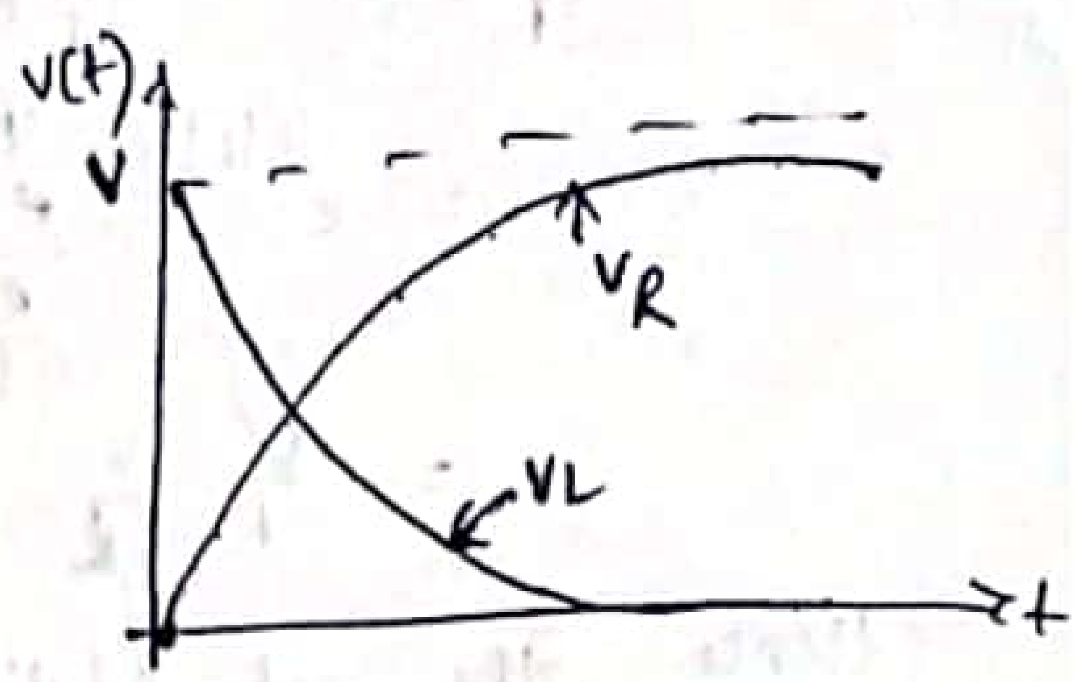


Note: Once the transient dies out within the short spell of time the steady state (st.st) current remains in the circuit is $\frac{V}{R}$.

From eq^n (3) we have two parts: (i) steady-state part (V/R) and (ii) Transient part $[V/R\ e^{-(R/L)t}]$

The time constant of a function $\left(\frac{V}{R}\right)e^{-(R/L)t}$ is the time at which the exponent of 'e' is unity, where e is the base of the natural logarithms. The term L/R is called the time constant and is denoted by τ.

$$\therefore \tau = \frac{L}{R} \text{ sec}$$



Voltage across Resistor

$$V_R = iR = R \times \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

$$= V\left[1 - e^{-(R/L)t}\right]$$

Voltage across the inductor is $V_L = L\frac{di}{dt}$

$$= L \cdot \frac{V}{R} \times \frac{R}{L} e^{-R/L\, t}$$

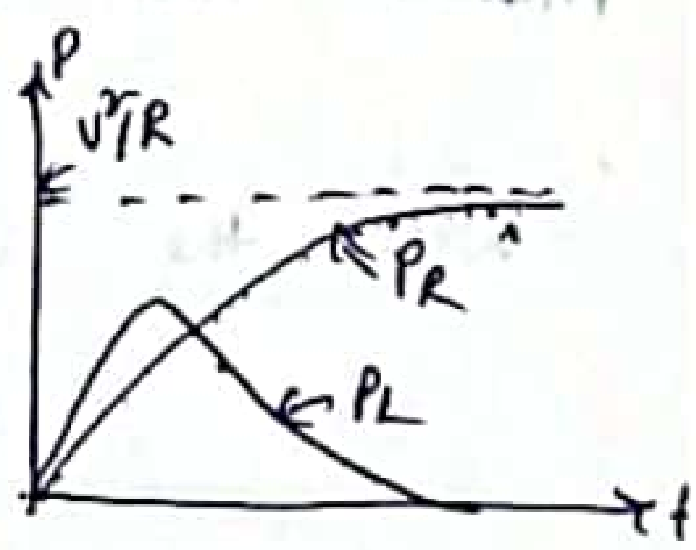
$$= V e^{-R/L\, t}$$

Power in Resistor $P_R = V_R i = V\left[1 - e^{-(R/L)t}\right]\left[1 - e^{-\left(\frac{R}{L}\right)t}\right] \cdot \frac{V}{R}$

$$= \frac{V^2}{R}\left[1 - 2e^{-(R/L)t} + e^{-\left(\frac{2R}{L}\right)t}\right] \quad \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$$

Power in inductor $P_L = V_L i = V\left[e^{-(R/L)t}\right] \times \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$

$$= \frac{V^2}{R}\left[e^{-\left(\frac{R}{L}\right)t} - e^{-\left(\frac{2R}{L}\right)t}\right]$$



[~~R-C~~ Series ~~circuit DC Excitation~~]:-

substitute $\tau = \frac{L}{R}$ in ③

$$\therefore i = \frac{V}{R}(1 - e^{-1}) = \frac{V}{R}(1 - 0.368) = 0.632 I_s \quad \left(\because I_s = \frac{V}{R}\right)$$

At time $\tau = \frac{L}{R}$ the current rises to 63.2% of final value.

$\tau = \frac{L}{R}$ = Time constant and $\frac{1}{\tau} = \frac{R}{L}$ = damping ratio.

$\therefore$ The R-L circuit rises to 63.2% of the final value

-Discharge case:-

Let us analyse another transient condition (natural response) of the R-L circuit. The circuit reaches at steady state (at $t=\infty$) and suddenly the voltage is withdrawn by opening the switch S and

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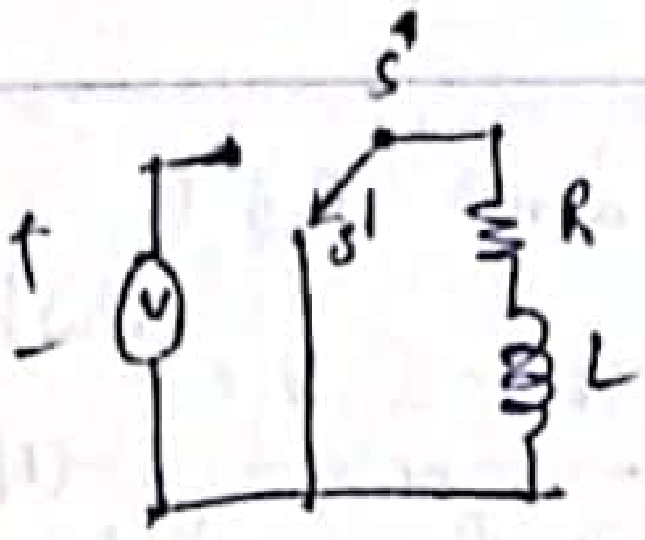
moving it to s'.



Apply KVL

$$Ri + L\frac{di}{dt} = 0$$

$$\left(D + \frac{R}{L}\right) i = 0 \rightarrow (1) \quad \left(\because D = \frac{d}{dt}\right)$$

Compare (1) with homogeneous differential equation $(D+P)x = 0$

where $P = \frac{R}{L}$; $x = i$, $k = 0$

particular integral $i_p = 0$

$\therefore$ Because we know $i_p = e^{-pt} \int e^{pt} \cdot k\,dt = e^{-pt} \int e^{pt} \cdot 0\,dt$

$$i_p = 0$$

$$\therefore \; i = i_c + i_p$$

$$= c^1 e^{-(R/L)t} + 0 \rightarrow (2)$$

At $t = 0^+$, the inductor will keep the steady state current $\left(\frac{V}{R}\right)$ even the switch is thrown to position s' short circuiting the charged R-L circuit and withdrawing the voltage source.

$$\therefore \; i(0^+) = i_{st \cdot st}(0^-) = \frac{V}{R}$$

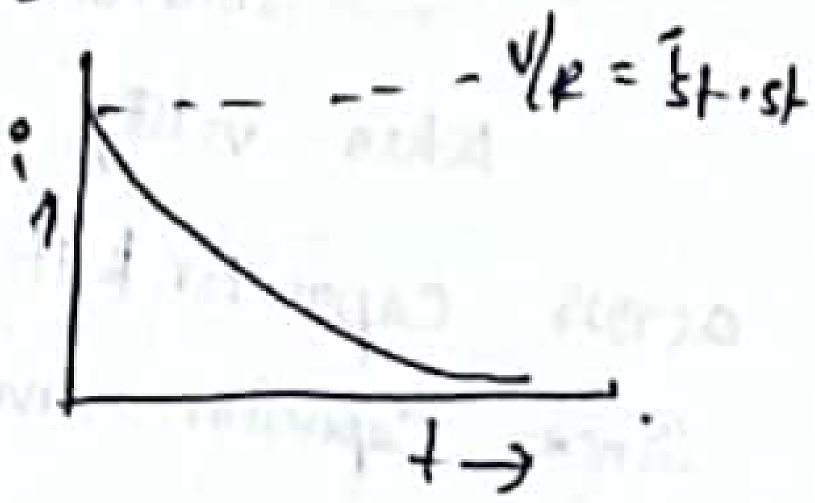
From (2) $\frac{V}{R} = c^1 e^{-R/L \cdot 0}$ $\left(\because t = 0, \; i(0^+) = \frac{V}{R}\right)$

$$c^1 = \frac{V}{R}$$

$\therefore$ The final solution becomes

$$i = \frac{V}{R} e^{-(R/L \cdot t)} \quad \left[\because I_0 = \frac{V}{R}\right]$$

$$= I_0 e^{-(R/L)t} \rightarrow (3) \text{ Amp}$$



From the above equation the current is exponentially decaying is shown in graph

Let $\tau = \frac{L}{R}$ from eq^n (3)

$$\therefore \; i = I_0 e^{-1} = 0.370 \; I_0$$

$\frac{L}{R}$ is called time constant is observed that the decaying current will reach to 37% of the initial steady state current at one time constant.

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Voltage across R & L

$$V_R = iR = V e^{-(R/L)t}$$

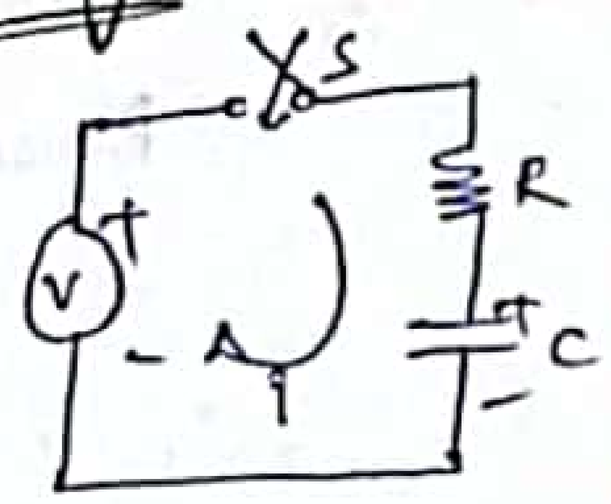
$$V_L = L\frac{di}{dt} = -V e^{-(R/L)t}$$

Power

$$P_R = V_R i = \frac{V^2}{R} e^{-2(R/L).t} \quad W$$

$$P_L = V_L i = -\frac{V^2}{R} e^{-2(R/L).t} \quad VA$$

Transient Response of RC Series Circuit having D.C excitation:



Let a d.c voltage V be applied (at t=0) to a series RC circuit, by closing a switch S in a series RC circuit, is shown in fig.

The current at t>0 being i, apply KVL

$$Ri + \frac{1}{C}\int i\,dt = V$$

Differentiation of above equation

$$\therefore R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\Rightarrow \left(D + \frac{1}{RC}\right) i = 0 \rightarrow ①$$

Compare eqn ① is a homogeneous differential equation $(D+P)x = 0$

where $P = \frac{1}{RC}$, $x = i$, $k = 0$

$i_p = 0$ since $k = 0$

$\therefore$ solution $i = i_c + i_p = i_c = C e^{-t/RC} \rightarrow ②$

When voltage applied initially there is no initial charge across capacitor but it acts as a short circuit causing current is V/R

Since capacitor never allows sudden changes in voltage (short circuit)

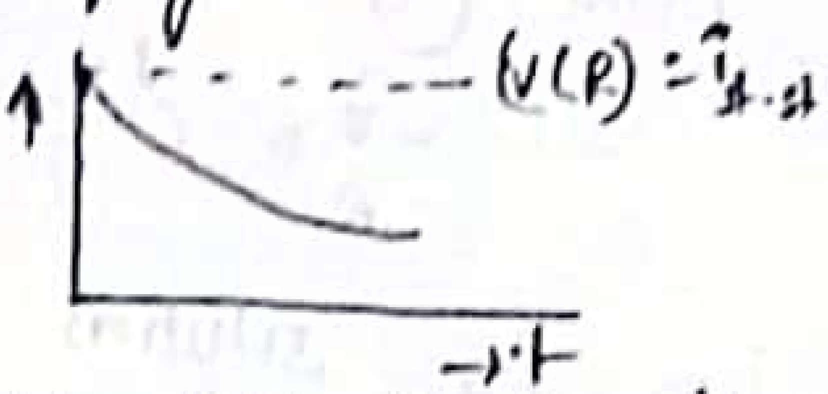
At $t = 0^+$, $i(0^+) = \frac{V}{R}$

From eqn ② $\frac{V}{R} = C \quad (\because e^0 = 1)$

Finally $i = \frac{V}{R} e^{-t/RC} \rightarrow ③$ Amp

From the above equation charging current decaying is shown in graph.

As the capacitor is getting charged, the charging current dies out.



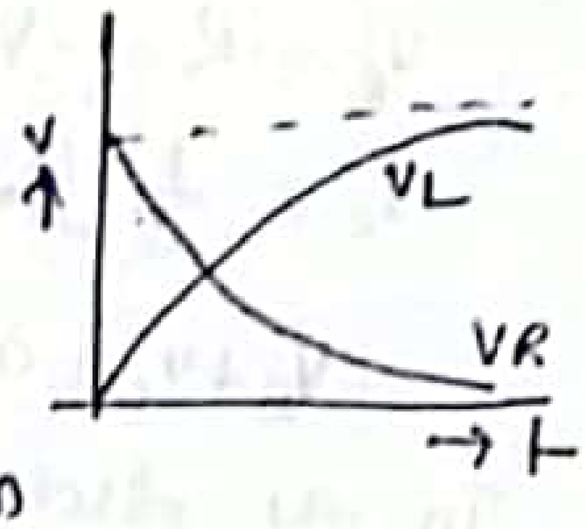
From the eq^n ③

i.e; The time constant of a function $\frac{V}{R} e^{-t/RC}$ is defined as the time at which the exponent of e is unity by τ ∴ $\boxed{\tau = RC}$ sec

Substitute in eq^n ① $\tau = RC$

<u>Voltage drops</u>

$$V_R = iR = V e^{-t/RC} \rightarrow \text{decaying function}$$



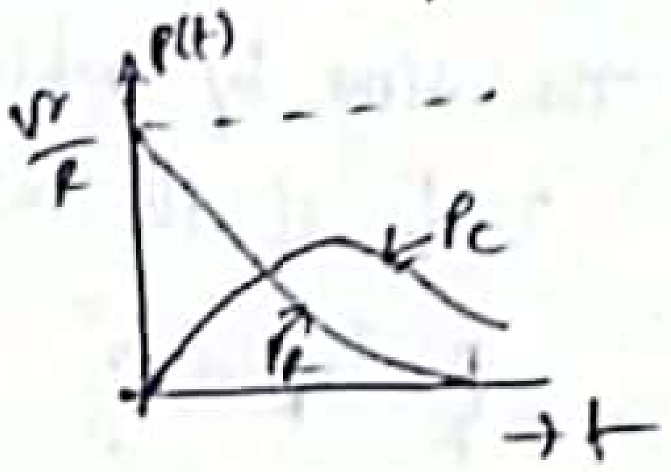
$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC}\,dt$$

$$= V(1-e^{-t/RC}) \rightarrow \text{Rising function}$$

Time constant is obtained by putting $\tau = RC$

$$\therefore V_C = V(1-0.368) = 0.632V$$

i.e; The time by which the capacitor attains 63.2% of steady state voltage

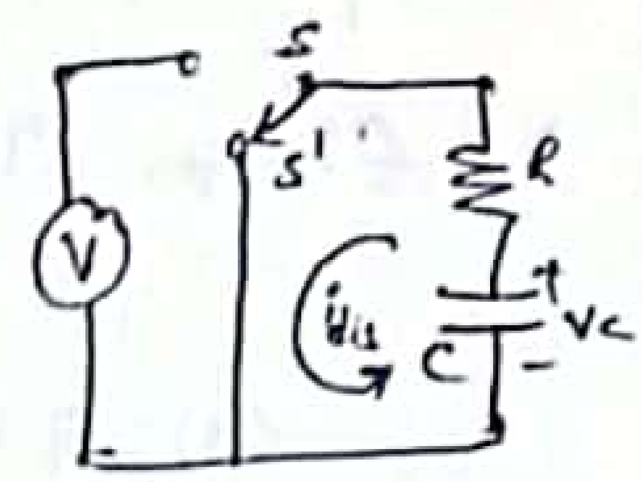


$$P_R = iV_R = \frac{V^2}{R} e^{-2\frac{t}{RC}} \quad W$$

$$P_C = iV_C = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right) \quad W$$

<u>Discharging Case</u>:-

when the switch S is thrown to contact 1,



then R-C circuit is shorted.

Apply KVL $Ri + \frac{1}{C}\int i\,dt = 0$

Differentiating the above equation

$$R\frac{di}{dt} + \frac{i}{C} = 0 \quad (\text{or}) \quad \left(D + \frac{1}{RC}\right) i = 0 \rightarrow ④$$

Compare ④ with homogenous equation

the $i = i_c \quad (\because i_p = 0)$

$$= c' e^{-t/RC} \rightarrow ⑤$$

At $t = 0^+$, the voltage across the capacitor will start discharging current through the resistor in opposite to original current direction i.e; direction -ve' and magnitude $\frac{V}{R}$.

$$i(0^+) = \frac{-V}{R}$$

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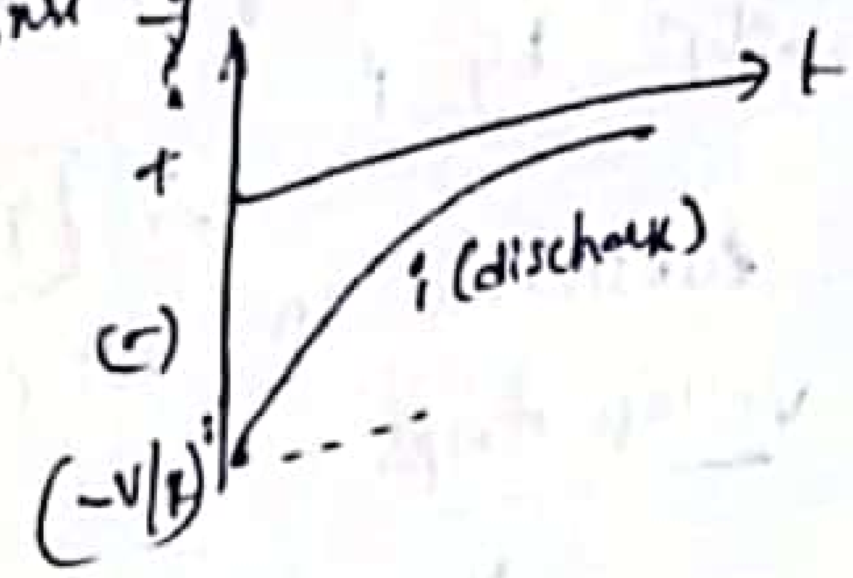
From (3)

$-\frac{V}{R} = c'$ (at $t=0^+$)

$\therefore$ solution $i = -\frac{V}{R} e^{-t/RC}$ A

From the above equation (natural response of the circuit) Current decay transient is shown in graph



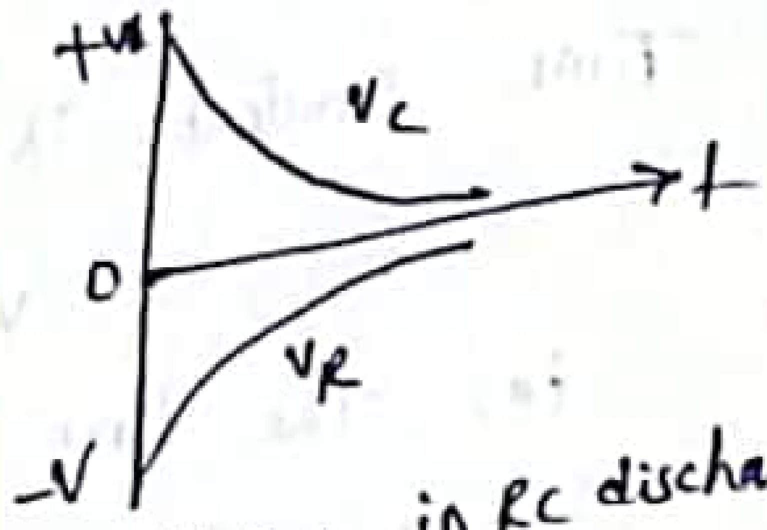
voltage drops:

$v_R = iR = -Ve^{-t/RC}$

$v_C = \frac{1}{C}\int i\,dt = Ve^{-t/RC}$

$v_R + v_C = 0$

In the discharging circuit, the time constant is the product of R & C



V_R & V_C in RC discharging circuit

$\therefore$ time constant $\tau = RC$

$\therefore v_C = Ve^{-1} = 0.369\,V$

$= 0.37\,V$

The time by which the capacitor discharges to 37% of its initial voltage

$P_R = iv_R = \frac{v^2}{R} e^{-2t/RC}$ W

$P_C = iv_C = -\frac{v^2}{R} e^{-2t/RC}$ W.

Charge stored in the capacitor during charging is

$q = Cv_C = CV(1-e^{-t/RC})$

(4) $q = Q(1-e^{-t/RC})$ $(\because Q = CV)$

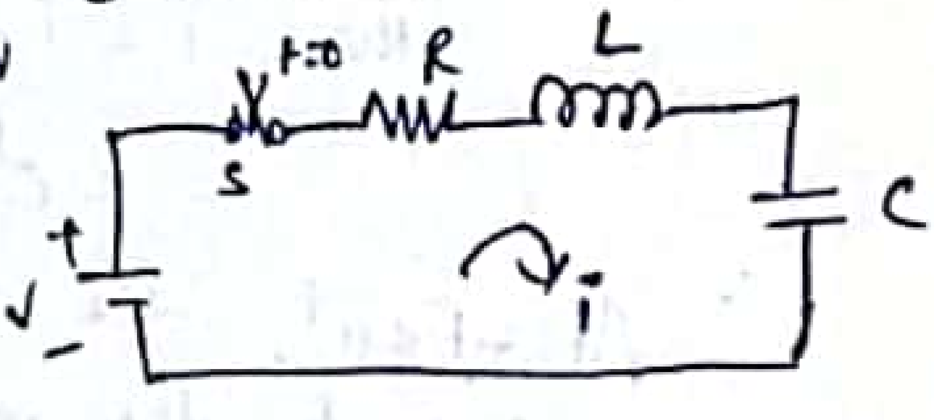
while during discharging

$q = Cv_C = CV\cdot e^{-t/RC}$ coulombs

$q = Qe^{-t/RC}$ columbs.

Transient response in Series RLC circuit with DC Excitation (second order circuit):-

In an RLC series circuit the capacitor and inductor are initially unchanged and are in series with a resistor. when the switch 's' is closed at $t=0$



Apply KVL

$\therefore\ v = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt$

Differentiating the above equation

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{c}i$$

By dividing both sides of the above equation, we can write with L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc}i = 0$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{Lc}\right)i = 0 \rightarrow (1)$$

Compare eq^n (1) with homogeneous second order differential equation $A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + cy = 0$

The characteristic equation is

$$D^2 + \frac{R}{L}D + \frac{1}{Lc} = 0,$$

The particular solution $i_p = 0$, $\therefore$ we have only i_c. The roots of above characteristic equation are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{Lc}}$$

Assuming $D_1 = \alpha + \beta$ where $\alpha = -\frac{R}{2L}$

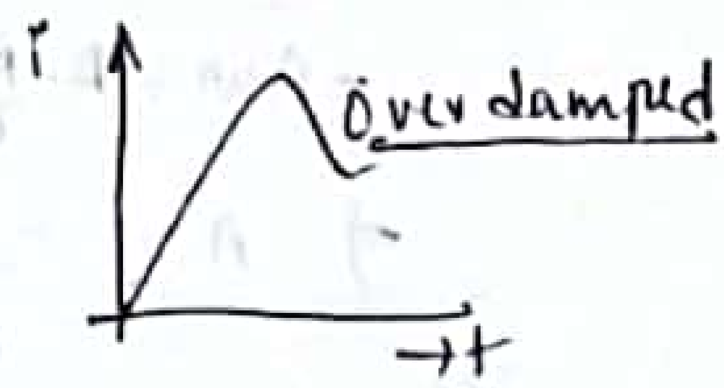
$D_2 = \alpha - \beta$ $\quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{Lc}}$

$\therefore$ Solution $i = c_1 e^{D_1 t} + c_2 e^{D_2 t}$, c_1, c_2 are constants

Case i) $\left(\frac{R}{2L}\right)^2 > \frac{1}{Lc}$; β is +ve & D_1, D_2 are real but unequal

$\therefore D_1 = \alpha + \beta$; $D_2 = \alpha - \beta$

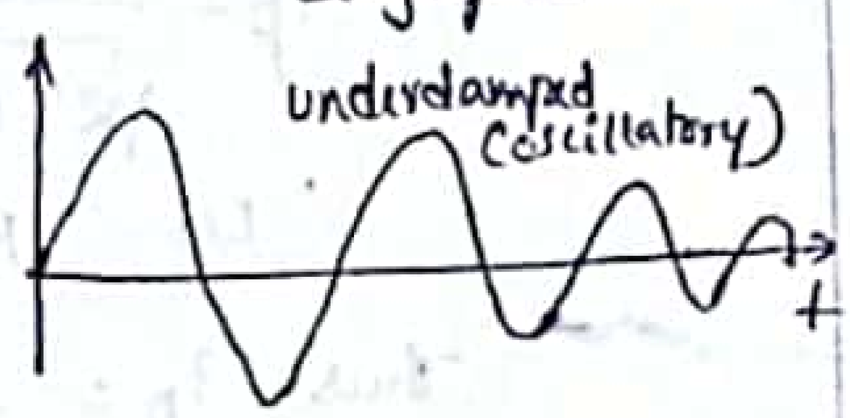
$$i = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$$
$$= e^{\alpha t}\left(c_1 e^{\beta t} + c_2 e^{-\beta t}\right)$$



Case (ii): $\left(\frac{R}{2L}\right)^2 < \frac{1}{Lc}$; β is imaginary, Roots D_1 & D_2 are complex Conjugate

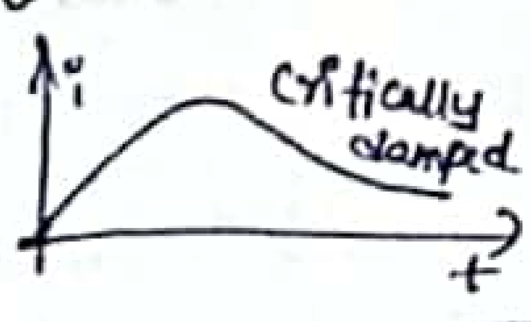
$D_1 = \alpha + j\beta$ $\quad i = c_1 e^{(\alpha+j\beta)t} + c_2 e^{(\alpha-j\beta)t}$

$D_2 = \alpha - j\beta$ $\quad = e^{\alpha t}\left[c_1 \cos(\beta t) + c_2 \sin(\beta t)\right]$

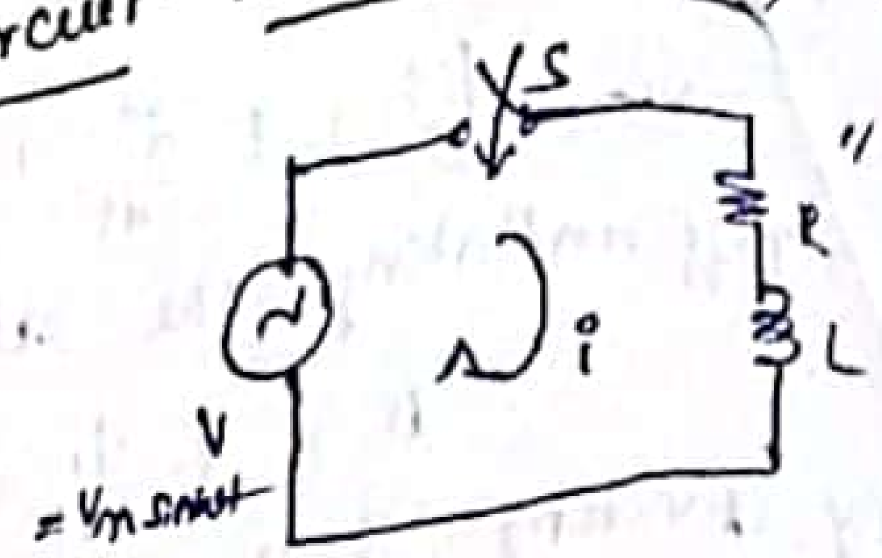


Case (iii): $\left(\frac{R}{2L}\right)^2 = \frac{1}{Lc}$; β is zero and D_1, D_2 is -ve & equal

$D_1 = D_2 = \alpha$ & $i = c_1 e^{\alpha t} + c_2 t e^{\alpha t} = e^{\alpha t}(c_1 + c_2 t)$



Transient Response in Series RL circuit with Sinusoidal



Let $v = V_m \sin(\omega t + \phi)$

ϕ varies from $0 - 2\pi$ depending on the switching instant

At $t = 0$ switch is closed

Apply KVL $\quad Ri + L\frac{di}{dt} = V_m \sin(\omega t + \phi)$

$$\frac{R}{L} i + \frac{d}{dt} i = \frac{V_m}{L} \sin(\omega t + \phi)$$

$$\left(D + \frac{R}{L}\right) i = \frac{V_m}{L} \sin(\omega t + \phi) \rightarrow (1)$$

Non homogeneous equation

Complementary function $i_c = C e^{-(R/L)t}$

Particular Solution $i_p = A\cos(\omega t + \phi) + B\sin(\omega t + \phi)$

First derivative of i_p is

$$Di_p = -A\omega \sin(\omega t + \phi) + B\omega \cos(\omega t + \phi)$$

Substitute i_p & Di_p in eq^n (1) $\quad [\because i = i_p]$

$$\Rightarrow \left[A\omega \sin(\omega t + \phi) + B\omega \cos(\omega t + \phi)\right] + \frac{R}{L}\left[A\cos(\omega t + \phi) + B\sin(\omega t + \phi)\right] = \frac{V_m}{L} \sin(\omega t + \phi)$$

$$\left(-A\omega + B\cdot\frac{R}{L}\right)\sin(\omega t + \phi) + \left(B\omega + A\cdot\frac{R}{L}\right)\cos(\omega t + \phi) = \frac{V_m}{L}\sin(\omega t + \phi)$$

Equating Coefficients of like terms

$$-A\omega + B\cdot\frac{R}{L} = \frac{V_m}{L} \qquad\qquad B\omega + A\cdot\frac{R}{L} = 0$$

$$\Rightarrow A = \frac{-\omega L V_m}{R^2 + \omega^2 L^2} \qquad\qquad \Rightarrow B = \frac{R V_m}{R^2 + \omega^2 L^2}$$

Substituting A & B in i_p equations

then $$i_p = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\left[-\omega L \cos(\omega t + \phi) + R\sin(\omega t + \phi)\right]$$

Let $R = \cos\theta$, $\omega L = \sin\theta$ & $\tan^{-1}\frac{\omega L}{R} = \theta$

Thus $$i_p = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\left[\sin(\omega t + \phi)\cos\theta - \cos(\omega t + \phi)\sin\theta\right]$$

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$$i = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin(\omega t+\phi-\theta) \quad [\because \sin(A-B) = \sin A\cos B - \cos A \sin B]$$

$$= \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \left[\sin(\omega t+\phi) - \tan^{-1}\frac{\omega L}{R}\right]$$

Net solution $i = i_c + i_p$

$$\therefore i = c\, e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t+\phi - \tan^{-1}\frac{\omega L}{R}\right).$$

Inductor will oppose any sudden change of current through it. As there was no current in the circuit before the switch was closed.

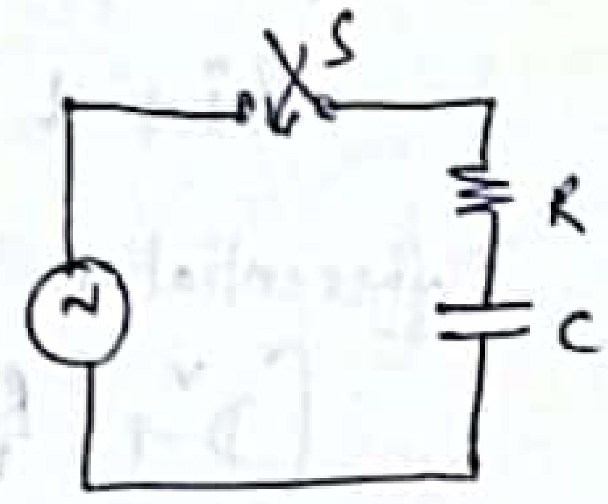
$i_0 = 0$ then at $t=0$,

$$i_0 = 0 = C + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left[\phi - \tan^{-1}\frac{\omega L}{R}\right]$$

$$\therefore i = e^{-(R/L)t}\left[-\frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left[\phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t+\phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right]$$

Final part is exponential factor $e^{-(R/L)t}$ which becomes zero at relatively short time as with increasing t.

Transient Response in Series RC circuit with Sinusoidal Excitation:-



$v = V_m \sin(\omega t+\phi)$

Apply KVL

$$Ri + \frac{1}{C}\int i\,dt = V_m \sin(\omega t+\phi)$$

Derivate the above equation then $\left[\frac{d}{dt}\cos\phi = -\sin\phi,\ \frac{d}{dt}\sin\phi = \cos\phi\right]$

$$Di + \frac{1}{RC} i = \frac{\omega V_m}{R} \cos(\omega t+\phi)$$

$$\left(D+\frac{1}{RC}\right)i = \frac{\omega V_m}{R}\cos(\omega t+\phi)$$

Solution $i = i_c + i_p$

$$i_c = c' e^{-t/RC}$$

$$i_p = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left[\omega t+\phi + \tan^{-1}\left(\frac{\omega L}{R}\right)\right] \quad [\because \text{From the similar above circuit (RL simplification) Simplification (similar)}]$$

$$\therefore i = c^{1} e^{-t/RC} + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

At $t=0$ the capacitor is short circuit

i_0 (initial current) $= \frac{V_m}{R}\sin\phi$

$$\therefore i_0 = \frac{V_m}{R}\sin\phi = c^{1} + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left[\phi + \tan^{-1}\frac{1}{\omega CR}\right]$$

(or) $$c^{1} = \frac{V_m}{R}\sin\phi - \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left[\phi + \tan^{-1}\frac{1}{\omega CR}\right].$$

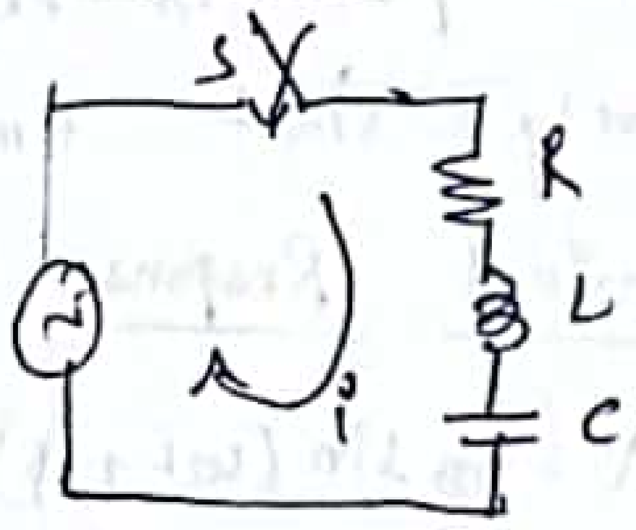
$$\therefore i = e^{-t/RC}\left[\frac{V_m}{R}\sin\phi - \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \sin\left[\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right]$$

first term is transient with decaying factor $e^{-t/RC}$.

Second term is steady state current which leads voltage by $\left[\tan^{-1}\frac{1}{\omega CR}\right.$

Transient Response in Series RLC with AC Excitation :-

$v = V_m \sin(\omega t + \phi)$



Apply KVL

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_m \sin(\omega t+\phi) \rightarrow (1)$$

Differentiation the above equation

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] i = \frac{\omega V_m}{L}\cos(\omega t + \phi) \rightarrow (2)$$

The particular solution can be obtained

Let $i_p = A\cos(\omega t + \phi) + B\sin(\omega t + \phi)$

$i_p' = -A\omega \sin(\omega t+\phi) + B\omega \sin(\omega t + \phi)$

$i_p'' = -A\omega^2 \cos(\omega t + \phi) - B\omega^2 \sin(\omega t + \phi)$

Substitution of values of i_p' and i_p'' in eq^n (2)

$$\left[-A\omega^2\cos(\omega t+\phi) - B\omega^2\sin(\omega t+\phi)\right] + \frac{R}{L}\left[-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)\right] + \frac{1}{LC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{\omega V_m}{L}\cos(\omega t+\phi).$$

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(9)

Comparing both sides, we get for sine coefficients

$$-B\omega^2 - A\frac{\omega R}{L} + \frac{1}{LC} = 0$$

$$A\left(\frac{\omega R}{L}\right) + B\left(\omega^2 - \frac{1}{LC}\right) = 0 \rightarrow (3)$$

and for cosine coefficients

$$-A\omega^2 + B\frac{\omega R}{L} + A\frac{1}{LC} = \frac{\omega V_m}{L}$$

$$+A\left[\omega^2 + \frac{1}{LC}\right] + B\left(\frac{\omega R}{L}\right) = \frac{\omega V_m}{L} - (4)$$

from (3) $A = -B\dfrac{\left(\omega^2 - \frac{1}{LC}\right)}{\frac{\omega R}{L}} = B\dfrac{\left(\frac{1}{LC} - \omega^2\right)}{\frac{\omega R}{L}}$

Substitution of A in eq^n (4)

$$B\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right] = \frac{\omega^2 R V_m}{L^2}$$

$$\therefore B = \frac{V_m \frac{\omega^2 R}{L^2}}{\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]} = \frac{V_m \frac{\omega^2 R}{L^2}}{\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

$$\text{Thus } A = B\frac{\left(-\omega^2 + \frac{1}{LC}\right)}{\frac{\omega R}{L}} = \frac{V_m \omega\left(\frac{1}{LC} - \omega^2\right)}{L\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

Here the values of A & B in i_p eq^n

$$i_p = \frac{V_m\omega\left(-\omega^2 + \frac{1}{LC}\right)}{L\left[\left(\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}\cos(\omega t + \phi) + \frac{V_m \frac{\omega^2 R}{L^2}}{\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}\sin(\omega t + \phi)$$

$$\text{Let } m\sin\theta = \frac{V_m\omega\left(-\omega^2 + \frac{1}{LC}\right)}{L\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

$$\text{and } m\cos\theta = \frac{V_m\frac{\omega^2 R}{L^2}}{\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

$$\text{Also } \frac{m\sin\theta}{m\cos\theta} = \tan\theta = \frac{\frac{V_m\omega\left(-\omega^2 + \frac{1}{LC}\right)}{L}}{V_m\frac{\omega^2 R}{L^2}} = \frac{L}{\omega R}\left(-\omega^2 + \frac{1}{LC}\right)$$

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$$\tan\theta = \frac{1}{R}\left(-\omega L + \frac{1}{\omega C}\right)$$

$$\theta = \tan^{-1}\left[\frac{1}{R}\left(-\omega L + \frac{1}{\omega C}\right)\right]$$

$\therefore$ $i_p = m \sin\theta \cos(\omega t+\phi) + m\cos\theta \sin(\omega t+\phi)$

$= m\left[\sin(\omega t+\phi)\cos\theta + \cos(\omega t+\phi)\sin\theta\right]$

$= m \sin(\omega t+\phi+\theta)$ $\quad [\because \sin A\cos B + \cos A \sin B = \sin(A+B)]$

$\Rightarrow i_p = m\left[\sin\left\{\omega t+\phi+\tan^{-1}\frac{1}{R}\left(\frac{1}{\omega C}-\omega L\right)\right\}\right]$

However $m = \sqrt{m^2\cos^2\theta + m^2\sin^2\theta}$

$$= \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}$$

$$i_p = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \cdot \sin\left[\omega t+\phi+\tan^{-1}\frac{\left(\frac{1}{\omega C}-\omega L\right)}{R}\right]$$

The Complementary function i_c being equal to the d.c response of RLC circuit.

<u>For overdamped case</u> :- when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

$$i = e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right) + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \sin\left[\omega t+\phi+\tan^{-1}\left(\frac{\frac{1}{\omega C}-\omega L}{R}\right)\right]$$

<u>For under damped case</u> :- when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$$i = e^{\alpha t}\left(C_1\cos\beta t + C_2\sin\beta t\right) + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \sin\left[\omega t+\phi+\tan^{-1}\frac{\left(\frac{1}{\omega C}-\omega L\right)}{R}\right]$$

<u>For critically damped case</u> :- when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

$$i = e^{\alpha t}(C_1+C_2 t) + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \sin\left[\omega t+\phi+\tan^{-1}\frac{\left(\frac{1}{\omega C}-\omega L\right)}{R}\right]$$

Laplace Transform :-

Let $f(t)$ be a function of time, then the value of function is zero for $t<0$ and it exists for $t>0$ and the function is continuous for all values of t from 0 to ∞. The Laplace transform of $f(t)$ may be

$$L[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st} dt$$

The Laplace transformation changes a function in time domain into a function in frequency domain.

Similarly, inverse Laplace transformation converts a function in frequency domain $F(s)$ to a function in time domain function $f(t)$. is

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi} \int_{-j}^{+j} F(s)\, e^{st} ds$$

Properties of Laplace transform

i) Linearity : $L[kf(t)] = k F(s)$

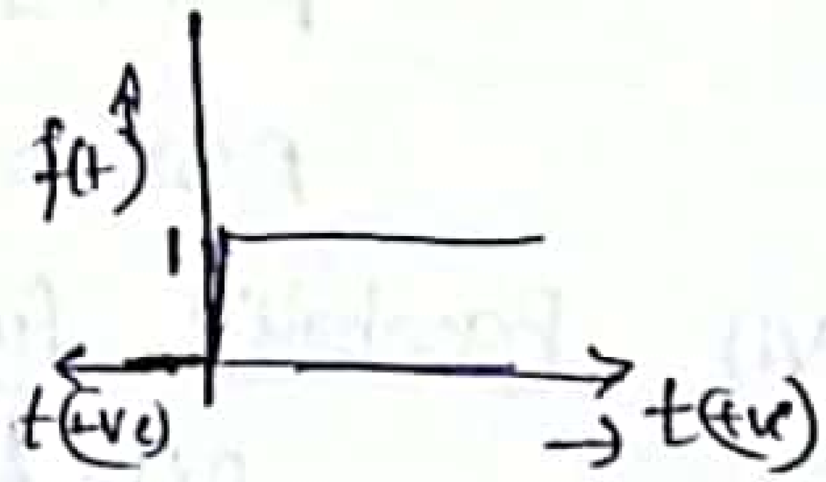
(ii) Superposition

$$L[f_1(t) + [f_2(t)] + f_3(t)] = F_1(s) + F_2(s) + F_3(s)$$

Laplace transform of some useful functions :-

i) Unit step function :- The most common driving function in electrical engineering is the unit step function

$$u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 0 \end{cases}$$



Laplace transformation can be

$$F(s) = L\,u(t) = \int_0^\infty e^{-st} dt$$

$$F(s) = \frac{1}{s}$$

(ii) Exponential function :- Exponential function being also very common in electric circuits $f(t) = e^{-\alpha t}$.

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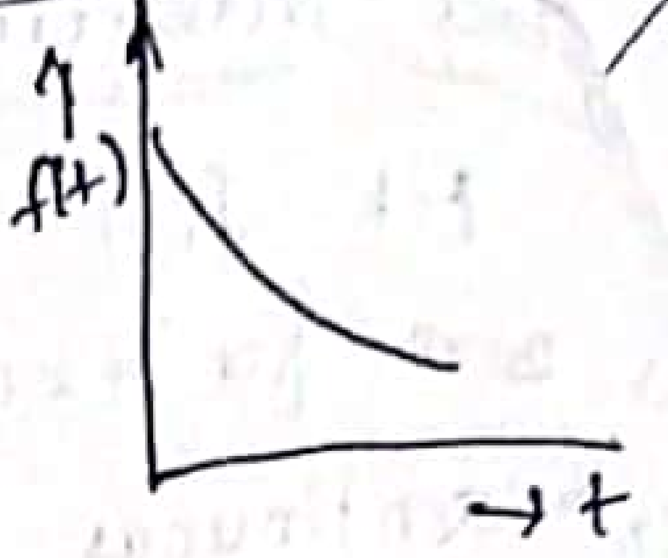
Laplace transform is



$$F(s) = L\,f(t) = L\,e^{-\alpha t}$$

$$= \int_0^\infty e^{-\alpha t}\, e^{-st}\, dt = \frac{1}{s+a}$$

(iii) Sinusoidal function : The function ~~of~~ extremely common for circuits encountering a.c source, the sinusoidal function

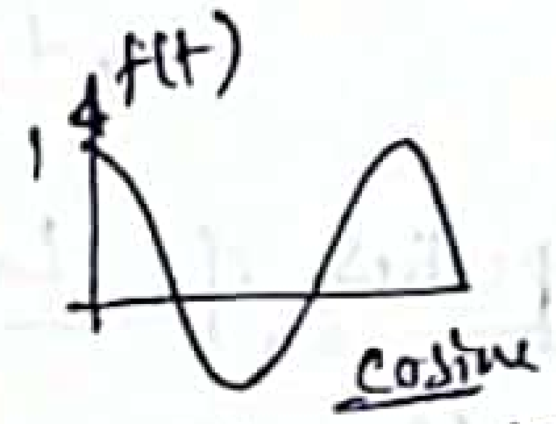
In time domain $f(t) = \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$

Laplace Transformation $F(s) = L\,f(t) = L\sin\omega t$

$$L\sin\omega t = \frac{1}{2j}\int_0^\infty e^{-(s-j\omega)t} - e^{-(s+j\omega)t}$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right] = \frac{\omega}{s^2+\omega^2}$$

(iv) ~~Cos~~ Cosinusoidal function :-

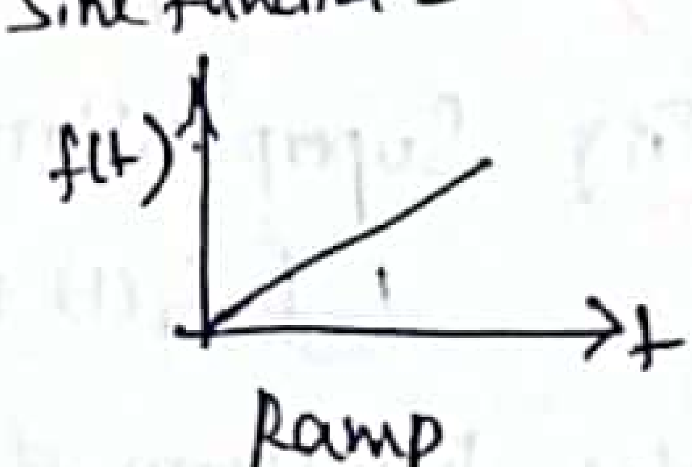


$$F(s)_{cos} = L\cos\omega t$$

$$= \frac{1}{\omega}\left[sF(s) - f(0^+)\right]$$

$$= \frac{1}{\omega}\left[s\frac{\omega}{s^2+\omega^2} - 0\right] \quad \left[\because \text{Sine function} = \omega/s^2+\omega^2\right.$$

$$= \frac{s}{s^2+\omega^2}$$



v) Ramp function :-

$$f(t) = t$$

$$F(s) = \int_0^\infty t\, e^{-st}\, dt$$

$$F(s) = \frac{1}{s^2}$$

vi) Parabolic function The parabolic function being

$$f(t) = ~~L~~ f(t)$$

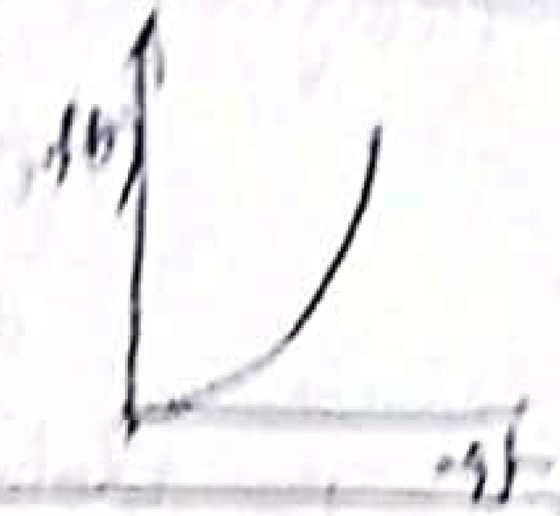
$$f(t) = L\int 2t\, dt$$

$$= \frac{F_1(s)}{s} + \frac{f^{1}(0^+)}{s} = \frac{F_1(s)}{s}$$

$$\therefore F_1(s) = L(2t) = \frac{2}{s^2}.$$

∴ Laplace transform of a parabolic function

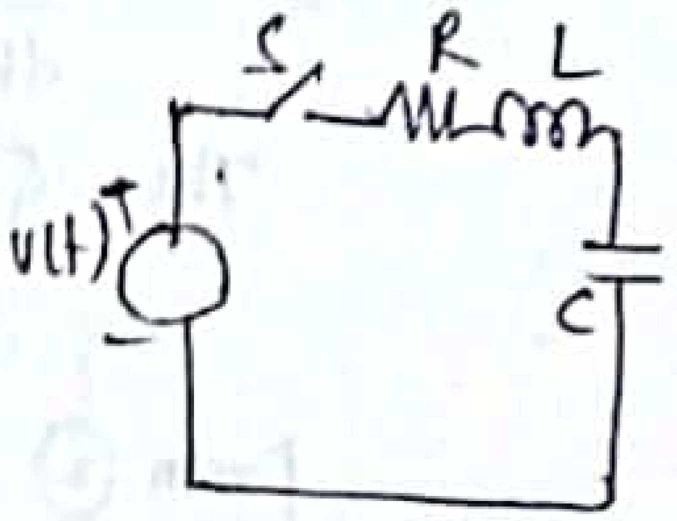
$$F(s) = \frac{2}{s^3}$$



Formulae:-

	Function	Laplace Transform
1	$u(t)$	$\frac{1}{s}$
2	e^{at}	$\frac{1}{s-a}$
3	e^{-at}	$\frac{1}{s+a}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
6	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
7	$e^{-at} f(t)$	$F(s+a)$
8	$e^{at} f(t)$	$F(s-a)$
9	$f(t-a)u(t-a)$	$e^{-as} F(s)$
10	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$
11	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$
12	$u_0(t)$ (or) $\delta(t)$	1

Response of Series R-L-C Circuit as Related to S-plane Location of Roots:-



Apply KVL

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = v(t)$$

differentiating the above equation and dividing by L

then $$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{1}{L}\frac{dv(t)}{dt} \rightarrow (1)$$

Homogenous equation is

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0 \rightarrow (2)$$

The solution for the above equation found by considering the characteristic equation as

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

The roots are

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{R}{LC}}$$

Let R_c = critical resistance

$$R_c = 2\sqrt{\frac{C}{L}}$$

Let η (zeta) be damping ratio given by the ratio of actual resistance to the critical resistance

$$\eta = \frac{R}{R_c} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

we know ω_n = undamped natural angular frequency.

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\therefore \; 2\eta\omega_n = 2\,\frac{R}{2}\sqrt{\frac{C}{L}} \cdot \frac{1}{\sqrt{LC}}$$

$$= \frac{R}{L}$$

$$\omega_n^2 = \frac{1}{LC}$$

Substituting $2\eta\omega_n$ for $\frac{R}{L}$ and ω_n^2 for $\frac{1}{LC}$ in (1), we get

$$\frac{d^2 i}{dt^2} + (2\eta\omega_n)\frac{di}{dt} + \omega_n^2 i = 0 \rightarrow (3)$$

The Quality factor Q of an RLC series circuit

$$Q = \frac{\omega_n L}{R}$$

From (2)

$$\frac{d^2 i}{dt^2} + \frac{\omega_n}{2}\frac{di}{dt} + \omega_n^2 i = 0$$

$$Q = \frac{1}{2\eta}$$

From (3)

$$s^2 + 2\eta\omega_n s + \omega_n^2 = 0$$

Roots are $s_1, s_2 = -\eta\omega_n \pm \omega_n\sqrt{\eta^2 - 1}$

General Solution is

$$i(t) = k_1 e^{(-\eta\omega_n + \omega_n\sqrt{\eta^2-1})t} + k_2 e^{(-\eta\omega_n - \omega_n\sqrt{\eta^2-1})t}$$

η varies 3 cases

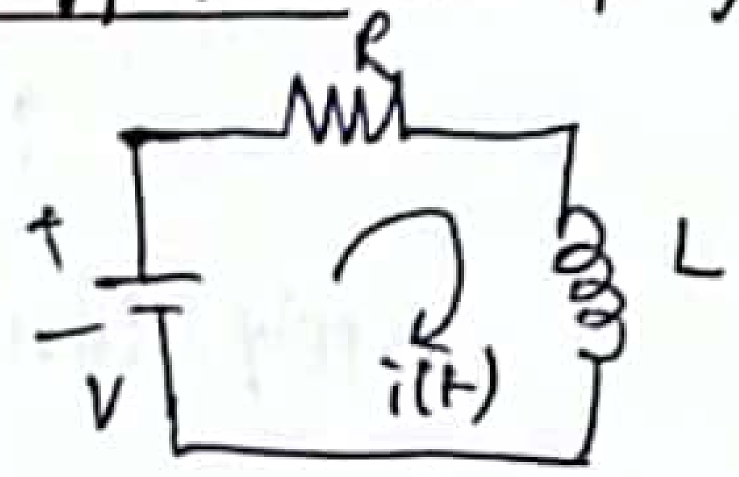
Case i) Underdamped circuit, $\eta < 1$, the natural roots are Complex and conjugate

Case (ii) Critically damped circuit :- $\eta = 1$
The nature of the roots are real and equal

Case (iii) Over-damped circuit $\eta > 1$ roots are real.

Response of R-L network for DC energy excitation (In Laplace):-

Consider an R-L network excited by a DC voltage as shown in fig



Apply KVL

$$R\, i(t) + L\frac{di(t)}{dt} = V$$

Applying Laplace Transform, we get.

$$L\left[R\, i(t) + L\frac{di(t)}{dt}\right] = L[V]$$

$$R\, i(s) + Ls\, i(s) - i(0) = \frac{V}{s}$$

$$i(s)(R+Ls) = \frac{V}{s} \quad (\because \text{ initial condition current } i(0^+) = 0)$$

$$i(s) = \frac{V}{s}\frac{1}{(R+Ls)} = \frac{V}{sR}\left[\frac{R}{R+Ls}\right]$$

$$= \frac{V}{R}\left[\frac{1}{s} - \frac{L}{Ls+R}\right] = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right]$$

Applying inverse Laplace transform:

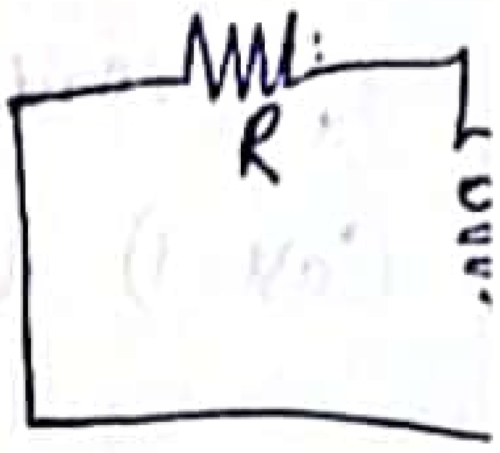
$$i(t) = L^{-1}\left[\frac{V}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)\right]$$

$$i(t) = \frac{V}{R}\left[1 - e^{-R/L\, t}\right]$$

Response is $i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$.

Response of R-L network for internal Energy Excitation:-

Consider the R-L network Excited by internal energy that is shown in fig.



Apply KVL

$$R\,i(t) + L\frac{di(t)}{dt} = 0$$

Apply Laplace Transform then

$$R\,i(s) + L\,s\,i(s) - L\,i(0^+) = 0$$

$$i(s)\left[\frac{R}{L} + s\right] = I_0 \quad (\because\ i(0^+) = I_0)$$

$$i(s) = \frac{I_0}{s+\frac{R}{L}}$$

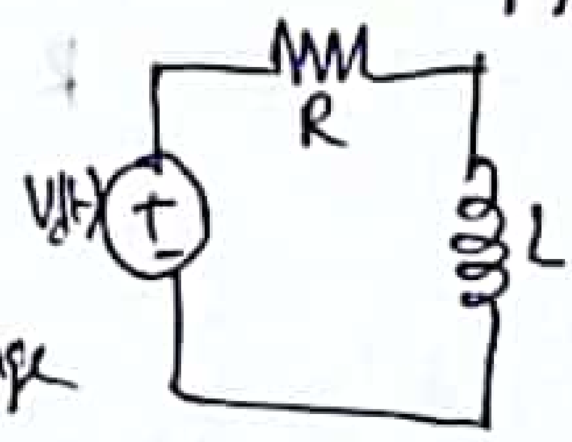
Apply inverse Laplace transform

$$i(t) = I_0\, L^{-1}\left[\frac{1}{s+\frac{R}{L}}\right]$$

$$i(t) = I_0\, e^{-R/L\, t}$$

Response of R-L network for unit impulse Excitation:-

Consider R-L network excited by unit impulse is shown in fig:



$\because$ at $t=0$ the inductor acts as open circuit, the entire impulse voltage appears across the inductor and hence the current in it will change instantaneously.

The current at $t=0^+$ is

$$i(0^+) = \frac{1}{L}\int_{0^-}^{0^+} V_0(t)\,dt$$

$$= \frac{1}{L} \qquad \left[\because \int_{0^-}^{0^+} v_0(t)\,dt = 1\right]$$

Apply KVL. $R\,i(t) + L\dfrac{di(t)}{dt} = 0$

Apply Laplace transform

$$\therefore R\,i(s) + L\,s\,i(s) - L\,i(0^+) = 0$$

$$i(s)\left[s+\frac{R}{L}\right] = \frac{1}{L} \Rightarrow i(s) = \frac{1}{L(s+\frac{R}{L})}$$

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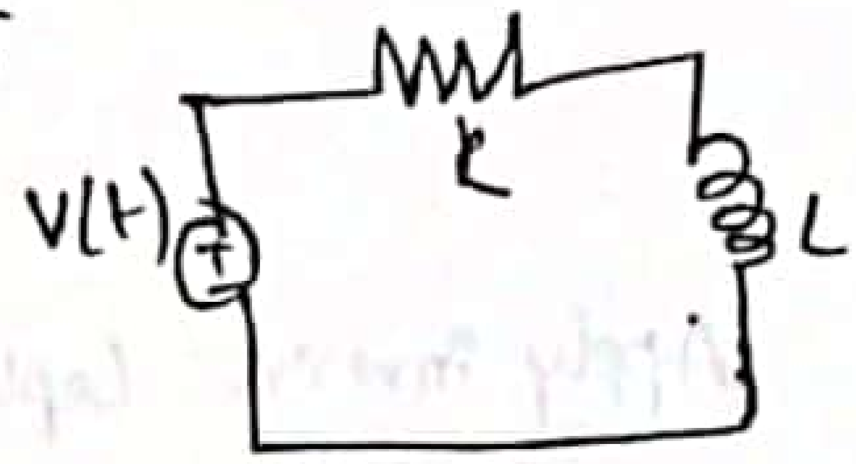
Applying inverse Laplace Transform, we get

$$i(t) = \frac{1}{L} e^{-\frac{R}{L}t}$$

Thus Response $i(t) = \frac{1}{L} e^{-\frac{R}{L}t}$.

Response of R-L network for unit step Excitation :-

Consider R-L network for unit step excitation is shown. The inductor acts as short circuit for $t=0^-$



$$i(0^-) = \frac{u(t)}{R}$$

At $t=0$, the initial current in the inductor is $i(0^+)=0$

$\therefore$ inductor acts as open circuit at $t=0^+$.

Apply KVL

$$R\, i(t) + L\frac{di(t)}{dt} = u(t) = 1$$

Applying Laplace transform we get

$$R\, i(s) + L s\, i(s) - L\, i(0^+) = \frac{1}{s}$$

$$i(s)\left[s + \frac{R}{L}\right] = \frac{1}{Ls} \Rightarrow i(s) = \frac{1}{Ls\left(s+\frac{R}{L}\right)}$$

Apply Inverse Laplace transform after splitting into partial fractions.

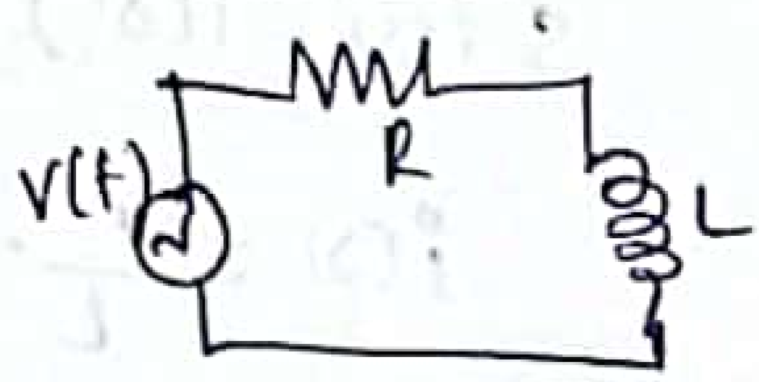
$$\therefore i(s) = \frac{1}{L}\left[\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right]\frac{L}{R} = \frac{1}{R}\left[\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right]$$

then from inverse Laplace

$$i(t) = \frac{1}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$

Response of R-L network for Sinusoidal Excitation :-

Consider the circuit is shown



$$v(t) = V_m \sin\omega t$$

Initial current in circuit $i(0^+) = 0$

Apply KVL

$$R\, i(t) + L\frac{di(t)}{dt} = V_m \sin\omega t$$

$$\frac{R}{L} i(t) + \frac{di(t)}{dt} = \frac{V_m}{L}\sin\omega t$$

Apply Laplace Transform

$$\frac{R}{L} i(s) + s\,i(s) - i(0^+) = \frac{V_m}{L} \frac{\omega}{s^2+\omega^2}$$

$$i(s) = \frac{V_m}{L} \cdot \frac{\omega}{s^2+\omega^2} \left[\frac{1}{s+\frac{R}{L}}\right]$$

$$i(s) = \frac{V_m}{L} \frac{\omega}{\omega^2+\frac{R^2}{L^2}} \left[\frac{1}{s+\frac{R}{L}} - \frac{s}{s^2+\omega^2} + \frac{R/L}{s^2+\omega^2}\right]$$

[From partial fraction]

Apply inverse Laplace Transform

$$i(t) = \frac{V_m}{L} \frac{\omega}{\omega^2+\frac{R^2}{L^2}} \left[e^{-R/L\,t} - \cos\omega t + \frac{R}{L\omega} \sin\omega t\right]$$

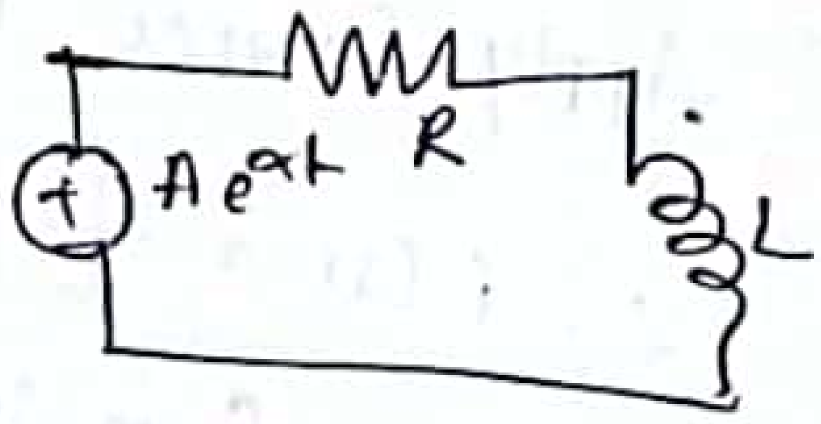
$$= \frac{V_m}{\omega^2 L^2 + R^2} \left[\omega L e^{-\frac{R}{L}t} - \omega L \cos\omega t + R \sin\omega t\right]$$

$$\therefore i(t) = \frac{V_m}{\sqrt{\omega^2L^2+R^2}} \left[\frac{\omega L}{\sqrt{\omega^2L^2+R^2}} e^{-\frac{R}{L}} + \sin(\omega t - \phi)\right]$$

where $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Response of R-L network for exponential excitation :-

Apply KVL



$$R\,i(t) + L\frac{di(t)}{dt} = A.e^{\alpha t}$$

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{A}{L} e^{\alpha t}$$

Apply Laplace Transform on both sides.

$$s\,i(s) - i(0^+) + \frac{R}{L} i(s) = \frac{A}{L} \cdot \frac{1}{s-a} \quad \left[\because e^{\alpha t} = \frac{1}{s-a}\right]$$

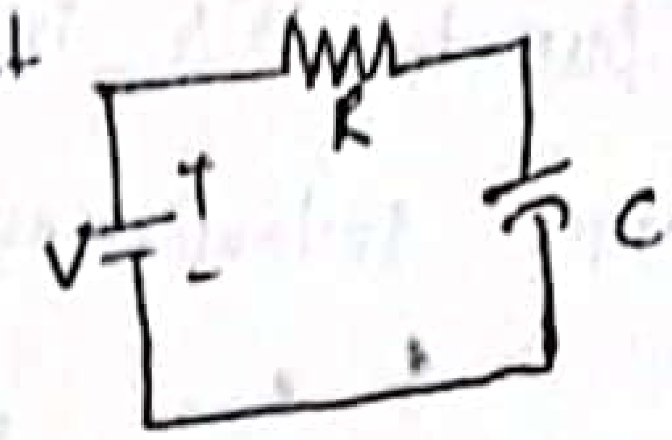
$$i(s) = \frac{A}{L} \cdot \frac{1}{s-\alpha} \cdot \frac{1}{s+\frac{R}{L}}$$

$$= \frac{A}{L} \cdot \frac{1}{\frac{R}{L}+\alpha} \left[\frac{1}{s-\alpha} - \frac{1}{s+\frac{R}{L}}\right]$$

Inverse Laplace

$$i(t) = \frac{A}{\alpha L+R} \left[e^{\alpha t} - e^{-\frac{R}{L}t}\right]$$

Response of RC network for DC voltage Excitation :-



At $t=0^+$, the capacitor acts as short circuit

and $i(0^+) = \frac{V}{R}$

Apply KVL to the Circuit

$$R\,i(t) + \frac{1}{C}\int i(t) = V$$

Differentiating the above expression

$$\frac{di(t)}{dt} + \frac{1}{RC}\, i(t) = 0$$

Apply Laplace Transform

$$S\,i(s) - i(0^+) + \frac{1}{RC}\, i(s) = 0$$

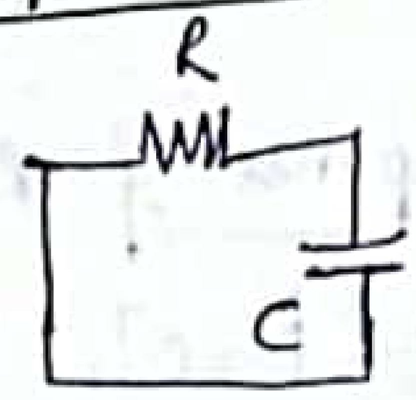
$$i(s) = \frac{V}{R}\cdot\frac{1}{S+\frac{1}{RC}}$$

Taking inverse Laplace Transform

$$i(t) = \frac{V}{R} e^{-t/RC}$$

The response is $i(t) = \frac{V}{R} e^{-t/RC}$

Response of R-C network for Internal Energy Excitation :-



Consider circuit, with capacitor initially

Charged at V_0.

At $t=0^+$, $i(0^+) = -\frac{V_0}{R}$

$\because$ $t=0^+$, Capacitor acts as short circuit in series with V_0.

Apply KVL

$$R\,i(t) + \frac{1}{C}\int i(t) = 0$$

differentiating the above equation w.r.to t.

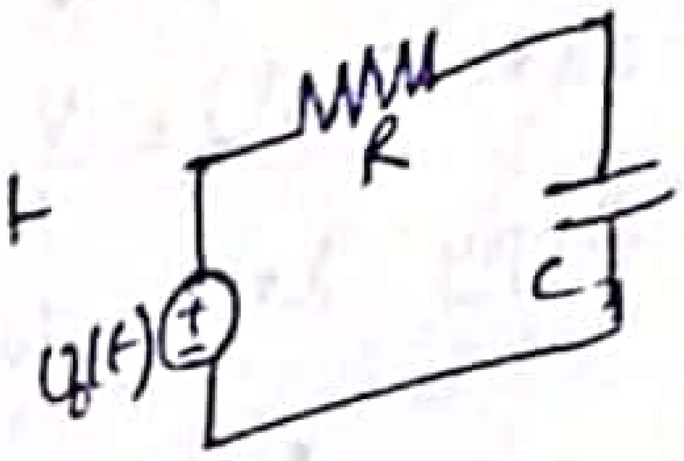
$$\frac{d\,i(t)}{dt} + \frac{1}{RC}\, i(t) = 0$$

Taking Laplace transform

$$i(t) = \frac{V_0}{R} e^{-\frac{1}{RC}t}$$

Response of R-C network for unit impulse Excitation :-

Since impulse current source is effective only during $t=0^-$ to $t=0^+$ and C tends to act like a short circuit during this time, the entire impulse current passes through the capacitor. Due to this impulse current, the voltage across the capacitor changes instantaneously and



$$v_c(0^+) = \frac{1}{C}\int_{0^-}^{0^+} U_0(t)\, dt$$

$$v_c(0^+) = \frac{1}{C} \Rightarrow i(0^+) = \frac{1}{RC}$$

Apply KVL for $t>0$, we obtain

$$Ri(t) + \frac{1}{C}\int i(t)\, dt = 0$$

Differentiating the above equation, we get

$$\frac{di(t)}{dt} + \frac{1}{RC} i(t) = 0$$

Apply Laplace Tranform

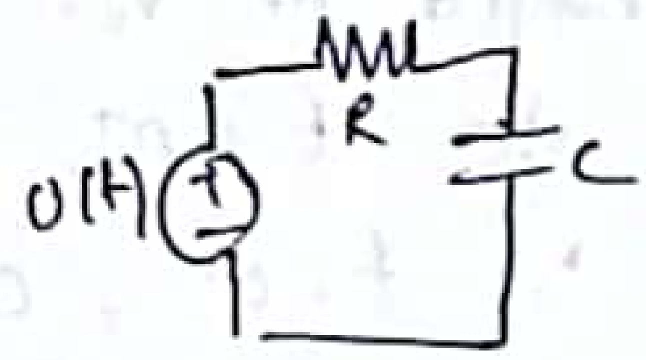
$$s i(s) - i(0^+) + \frac{1}{RC} i(s) = 0$$

$$i(s) = \frac{1}{RC} \cdot \frac{1}{s+\frac{1}{RC}}$$

Taking inverse Laplace Transform, we get

$$i(t) = \frac{1}{RC} e^{\frac{-1}{RC}t}$$

Response of R–C network for unit step Excitation:



At $t=0^+$, $i(0^+) = \frac{U(t)}{R} = \frac{1}{R}$

$\because$ $U(t)=1$ for $t>0$ and

Capacitor acts as short circuit

Apply KVL

$$R\, i(t) + \frac{1}{C}\int i(t)\, dt = U(t) = 1$$

Differentiating the above equation

$$\frac{d\, i(t)}{dt} + \frac{1}{RC} i(t) = 0$$

Taking Laplace Transform

$$s\, i(s) - i(0^+) + \frac{1}{RC} i(s) = 0$$

$$i(s)\left[s+\frac{1}{RC}\right] = \frac{1}{R} \Rightarrow i(s) = \frac{1}{R} \cdot \frac{1}{s+\frac{1}{RC}}$$

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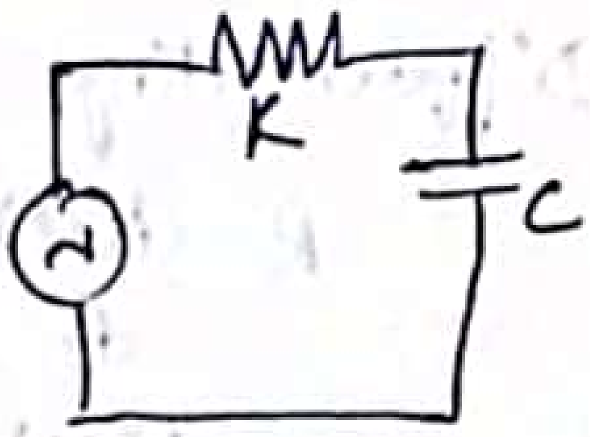
Taking Inverse Laplace transform

$$i(t) = \frac{1}{R} e^{-\frac{1}{RC}t}.$$

Response of R-C network for sinusoidal Excitation :-

$V(t) = V_m \sin\omega t$



At $t = 0^+$, $i(0^+) = \frac{V_m \sin\omega t}{R} = 0$ $(\because t = 0^+)$

Apply the KVL

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = V_m \sin\omega t$$

Differentiating the above equation w.r.to t,

$$R\frac{di(t)}{dt} + \frac{1}{C} i(t) = V_m \omega \sin\omega t$$

$$\frac{di(t)}{dt} + \frac{1}{RC} i(t) = \frac{V_m \omega \cos\omega t}{R}.$$

Apply Laplace transform

$$S\,i(s) - i(0^+) + \frac{1}{RC} i(s) = \frac{V_m \omega}{R}\cdot\frac{S}{S^2+\omega^2}$$

$$i(s) = \frac{V_m\omega}{R}\cdot\frac{S}{S^2+\omega^2}\cdot\frac{1}{S+\frac{1}{RC}}$$

Splitting into partial fractions

$$i(s) = \frac{V_m\omega}{R\left[\left(\frac{1}{RC}\right)^2+\omega^2\right]}\left[\frac{\frac{-1}{RC}}{S+\frac{1}{RC}} + \frac{\frac{1}{RC}S}{S^2+\omega^2} + \frac{\omega^2}{S^2+\omega^2}\right]$$

Taking inverse Laplace transform

$$i(t) = \frac{V_m\omega}{R\left[\left(\frac{1}{RC}\right)^2+\omega^2\right]}\left[\frac{-1}{RC}e^{-\frac{1}{RC}t} + \frac{1}{RC}\cos\omega t + \omega\sin\omega t\right]$$

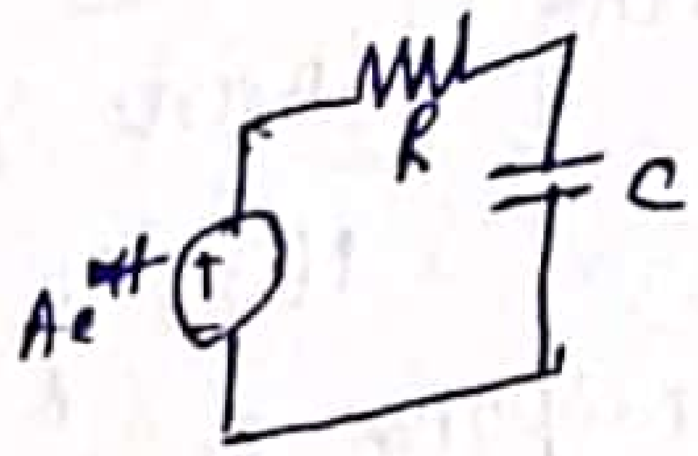
$$= \frac{-V_m\omega}{\omega C\left(R^2+\frac{1}{\omega^2C^2}\right)} e^{-\frac{1}{RC}t} + \frac{V_m}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\sin(\omega t+\phi)$$

Response of R-C Network for exponential Excitation :-

Consider the R-C network for Exponential is shown in fig

$V(t) = A e^{\alpha t}$

At $t=0^+$, $i(0^+) = \frac{Ae^{\alpha t}}{R} = \frac{A}{R}$



Apply KVL

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = Ae^{\alpha t}$$

Differentiating the above equation

$$R\frac{di(t)}{dt} + \frac{1}{C}i(t) = A\alpha e^{\alpha t}$$

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = \frac{A\alpha e^{\alpha t}}{R}$$

Taking Laplace transforms, we obtain

$$s\,i(s) - i(0^+) + \frac{1}{RC}i(s) = \frac{A\alpha}{R}\cdot\frac{1}{s-\alpha}$$

$$i(s) = \left[\frac{A\alpha}{R}\frac{1}{s-\alpha} + \frac{Ae^0}{R}\right]\left[\frac{1}{s+\frac{1}{RC}}\right]$$

$$= \left[\frac{A\alpha}{R}\frac{1}{s-\alpha}\times\frac{1}{s+\frac{1}{RC}} + \frac{A}{R}\frac{1}{s+\frac{1}{RC}}\right]$$

$$= \frac{A\alpha}{R}\left[\frac{1}{s-\alpha} - \frac{1}{s+\frac{1}{RC}}\right]\left[\frac{1}{\alpha+\frac{1}{RC}}\right] + \frac{A}{R}\frac{1}{s+\frac{1}{RC}}$$

Inverse Laplace

$$i(t) = \left[\frac{A}{R} - \frac{A}{R+\frac{1}{\alpha C}}\right]e^{-\frac{1}{RC}t} + \frac{Ae^{\alpha t}}{R+\frac{1}{\alpha C}}$$